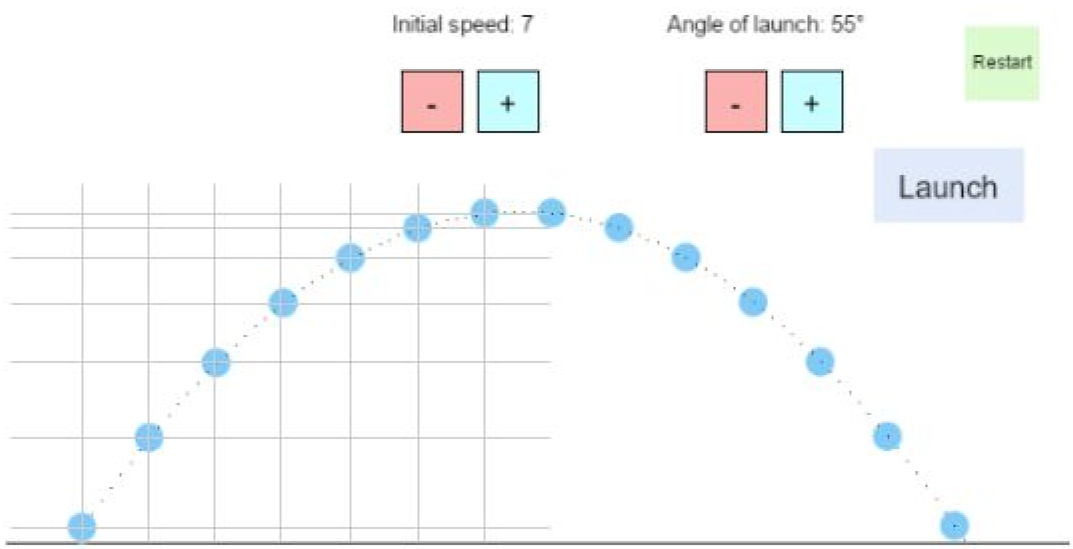
Projectile Motion

1) The important point of projectile motion is that it is a motion in 2 dimension with an acceleration only in one direction (vertical due to gravity). One way to make this explicit to the student can be done as follow:

Set the speed to 7 and the angle of launch to 55 degrees.

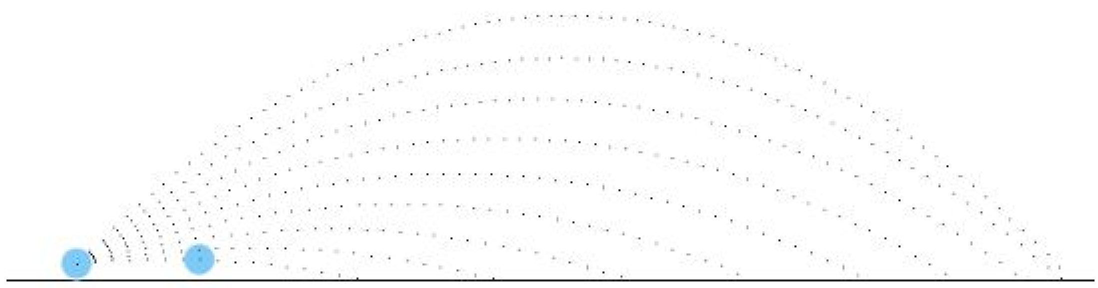
Ask the students to compare the change in the horizontal position between each time interval. How does it compare to the change in the vertical position?
The diagram below makes this quite explicit. The acceleration due to gravity only acts vertically, therefore only the vertical component of the velocity changes. The horizontal component of the velocity remains constant, so for each unit time the horizontal distance travelled is the same.
A comparison between the vertical motion and free-fall can be made.



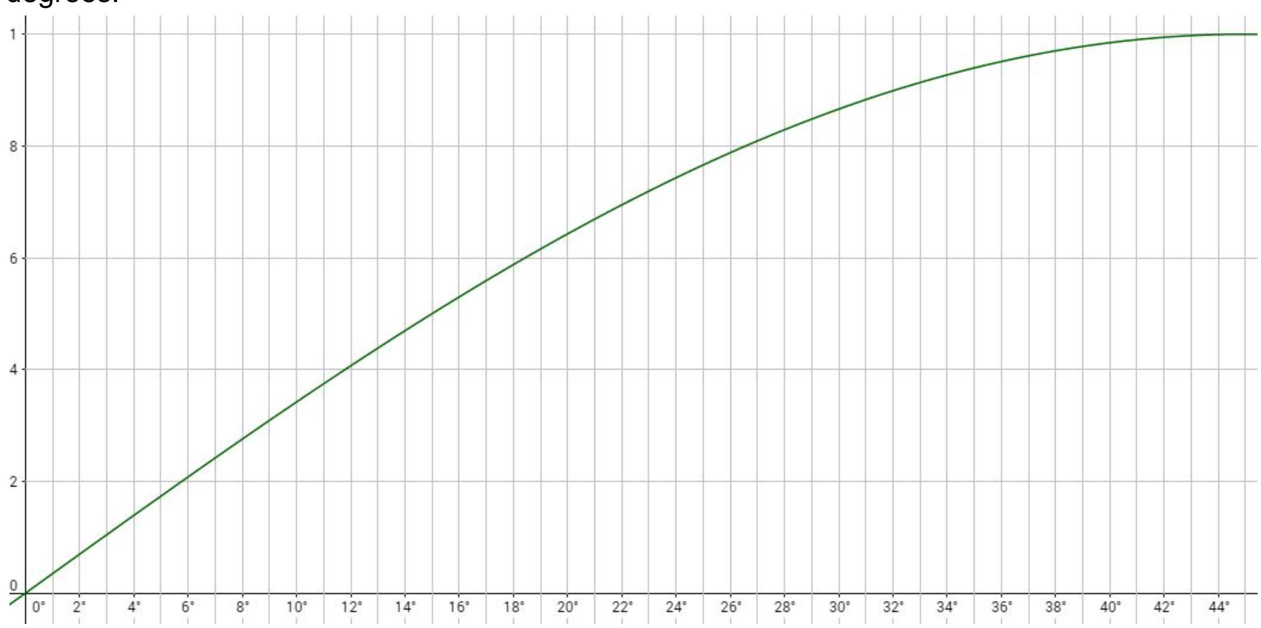
2) Students can be asked to find the angle that gives the maximum range for a given velocity using the animation. The same can be done by using the equation of motion.
It can be shown that the range is given by the following equation:
$x = \frac{u^2 sin(2\theta)}{g}$ where x is the range (maximum horizontal displacement), u the initial speed, θ the angle of launch (the angle that the velocity vector makes with the horizontal) and g the acceleration due to gravity.

An interesting connection can be made to trigonometry and possibly calculus here.

Ask the students to launch the ball at 45 degrees with a speed of 7, now ask the students to decrease the angle by 5 degrees keeping the initial speed constant. Repeat until the angle of launch cannot be reduced further. The following diagram will be obtained.



Every time the angle was changed by the same amount, however the range changes by a different amount every time. There is very little change in the range between 45 degrees and 50 degrees but as the angle decreases it seems to have a larger impact on the range.
The graph below show the graph for the equation f(x) = sin(2x) for angles between 0 and 45 degrees.



It can be seen that the curve is the steepest around x=0 and as x increases the slope of the tangent decreases (the curve becomes less steep).
How does this connect with the previous observation?

Would a constant change in the speed for a constant angle of launch have the same effect on the range? (How does the gradient of f(x) = x^2 changes as x increases?)